# RELIABILITY PROBLEMS AND PARETO-OPTIMALITY IN COGNITIVE RADAR (INVITED PAPER)


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## ABSTRACT

Cognitive radar refers to an adaptive sensing system exhibiting high degree of waveform adaptivity and diversity enabled by intelligent processing and exploitation of information from the environment. The next generation of radar systems are characterized by their application to scenarios exhibiting non-stationary scenes as well as interference caused by use of shared spectrum. Cognitive radar systems, by their inherent adaptivity, seem to be the natural choice for such applications. However, adaptivity opens up reliability issues due to uncertainties induced in the information gathering and processing. This paper lists some of the reliability aspects foreseen for cognitive radar systems and motivates the need for waveform designs satisfying different metrics simultaneously towards enhancing the reliability. An iterative framework based on multi-objective optimization is proposed to provide Pareto-optimal waveform designs.



***Index Terms*—** Cognitive radar, reliability, waveform diversity, waveform optimization, multi-objective optimization, Pareto-optimal design.


## 1. INTRODUCTION

Signal processing and design for radar has been of interest to engineers, system theorists and mathematicians in the last couple of decades. In the last decade, however, the radar world has been revolutionized by significant increase in the computational resources; an ongoing revolution with considerable momentum [1, 2]. Such advances are enabling waveform design and processing schemes that can be cognitive (also referred to as adaptive, or smart) while being extremely agile in modifying information collection strategy based on new measurements, and/or modified target or environmental parameters. These novel design and processing schemes have also opened new avenues for enhancing robustness in radar detection/estimation, as well as coexistence in networked environments with limited resources such as a shared spectrum— all leading to increased reliability.

Waveform design and processing for radar has a crucial role particularly in fulfilling the above promises of adaptivity, agility and reliability: it is widely known that a judicious design of the transmit waveforms can significantly improve the performance of active radar systems. However, the waveform design usually deals with various measures of quality (including detection/estimation and information-theoretic criteria), and moreover, the practical condition that the employed signals must belong to a limited signal set. Such diversity of design metrics and signal constraints lays the ground for many interesting research works in waveform optimization. Additionally, efficient waveform design for next-generation radar is a topic of great interest due to the recent growing demands in increasing the number of antennas/sensors in different radar applications (motivated by recent advances on MIMO radar). From a similar viewpoint, efficient algorithms for signal processing are necessary once the backscattered signals are collected from the surrounding environment [3].

Cognitive radar represents a class of remote sensing systems capable of intelligent interaction with the environment by adapting both waveform and processing functions based on contextual awareness [4]. Its inherent ability to adapt, seems to be an attractive choice for next- generation radar systems that need to cope with dynamic situations with agility and reliability. This overview paper summarizes the reliability aspects of cognitive radar systems and describes approaches to reliable waveform design. The various waveform design metrics are reviewed in Section II, reliability aspects are highlighted in Section III and algorithm design is provided in Section IV where in Pareto-optimal designs are also included.

## 2. WAVEFORM QUALITY METRICS

We consider a cognitive radar system employing $N$ (temporally or spatially diverse) measurements in a scenario involving a single target. The resulting received signal corresponding to the range-Doppler cell of interest may be written as[1]

$$\boldsymbol{r} = \boldsymbol{\Lambda}_{\boldsymbol{\theta}} \boldsymbol{x} + \boldsymbol{c}(\boldsymbol{x}) + \boldsymbol{\eta}, \tag{1}$$

[1]We use bold lowercase letters for vectors and bold uppercase letters for matrices. Please see Table 1 for other notations used throughout the paper.

**Table 1**. Notations

| | |
|---|---|
| $\boldsymbol{x}(k)$ | the $k^{th}$ entry of the vector $\boldsymbol{x}$ |
| $\Vert\boldsymbol{x}\Vert_n$ | the $l_n$-norm of $\boldsymbol{x}$, defined as $(\sum_k \vert\boldsymbol{x}(k)\vert^n)^{\frac{1}{n}}$ |
| $\boldsymbol{X}^H$ | the complex conjugate of a matrix $\boldsymbol{X}$ |
| $\boldsymbol{X}^T$ | the transpose of a matrix $\boldsymbol{X}$ |
| $\mathrm{tr}(\boldsymbol{X})$ | the trace of a matrix $\boldsymbol{X}$ |
| $\boldsymbol{I}_n$ | the identity matrix of dimension $n$ |

where, $\boldsymbol{\theta}$ contains features attributed to the target (direction of arrival, Doppler, frequencies of operation etc) and $\boldsymbol{\Lambda_\theta}$ is an $N \times N$ matrix comprising target characteristics, Doppler effects and the propagation environment for each of the dimensions. Further, $\boldsymbol{x}$ is the $N$-dimensional transmit signal, $\boldsymbol{c}(\boldsymbol{x})$ represents the signal-dependent clutter, and $\boldsymbol{\eta}$ is the receiver front-end noise (typically additive white Guassion noise, AWGN). The components can be statistically characterised, without loss of generality, as

$$\begin{aligned} E[\boldsymbol{\eta}] = \boldsymbol{0}, \quad E[\boldsymbol{\eta}\boldsymbol{\eta}^H] = \boldsymbol{R}_{nn}, \\ E[\boldsymbol{c}(\boldsymbol{x})] = \boldsymbol{0}, \quad E[\boldsymbol{c}(\boldsymbol{x})\, \boldsymbol{c}^H(\boldsymbol{x})] = \boldsymbol{R}_{cc}. \end{aligned} \tag{2}$$

Note that such a model subsumes several basic models earlier developed in the literature: for example, the model in [5, 6] is obtained by choosing $\boldsymbol{\Lambda_\theta}$ being a diagonal matrix with the $(k, k)$ diagonal entry being $\alpha e^{j(k-1)\nu}$, with $\nu$ being the normalized Doppler shift. For a MIMO uniform linear array with spacing of $d$, these entries would also involve the direction of arrival, with the $(k, k)$ (diagonal) entry given as $e^{j(k-1)\theta d/\lambda}$.

While several ingredients constitute a cognitive radar system [4], one of the most critical aspects (and thus our focus in the sequel) would on the design of probing waveforms that make a cognitive information-collection possible. To this end, we begin with a review of quality metrics for the *waveform* $\boldsymbol{x}$, to be designed for the system abstracted in (1). Some traditional performance metrics for designing $\boldsymbol{x}$ include the correlation and ambiguity functions [1,7]. We note that, depending on the sensing scenario, such metrics are not always the best measures of quality for the probing waveforms. In addition, due to the large number of constraints arising from the two-dimensional nature of the ambiguity function [1, 8], designing $\boldsymbol{x}$ for a given ambiguity pattern is deemed to be difficult. However, there are a number of tractable metrics leading to *practical* designs; see below.

### 2.1. Correlation Properties

Sequences with good autocorrelation properties have been considered in radar application towards achieving an enhanced performance in the presence of clutter. To design such sequences, minimizing the sidelobe levels of the correlation function have been considered [9, 10]. Denoting $\boldsymbol{r}$ to be the $(2N-1)$-length conjugate symmetric correlation of $\boldsymbol{x}$ [9, 10], an useful approach is to minimize the *weighted integrated sidelobe level* of the correlation, defined as

$$W_{ISL}(\boldsymbol{x}) = \sum_{k=1}^{N-1} \boldsymbol{p}(k)|\boldsymbol{r}(k)|^2, \tag{3}$$

where $\boldsymbol{p}(k)$ are the nonnegative weights to be assigned by the user/system. Alternatively, one may consider a minimization of the *peak sidelobe level* of the correlation function, viz.

$$PSL(\boldsymbol{x}) = \max\ \{|\boldsymbol{r}(k)|\}_{k=1}^{N-1}. \tag{4}$$

### 2.2. Spectral Properties

Spectral properties have a strong connection (established by the Fourier transform) with correlation properties [10]. Nevertheless, it is sometimes more interesting to focus on the spectral properties of the probing waveforms. In wideband applications, for instance, spectral shaping of the waveform becomes important when nulls in specific frequencies are desired to avoid interference [11]. Such a design objective involves minimizing the Spectral Error Measure (SEM),

$$SEM(\boldsymbol{x}) = \|\boldsymbol{F}(\Omega)\boldsymbol{x} - \boldsymbol{\Xi}\boldsymbol{y}\|^2, \tag{5}$$

with respect to $\boldsymbol{x}$ and $\boldsymbol{\Xi}$, where $\boldsymbol{\Xi}$ is a $N \times N$ diagonal matrix including auxiliary *phasors* on the diagonals, $\boldsymbol{F}(\Omega)$ denotes the rows of the DFT matrix indexed by $\Omega$ and $\boldsymbol{y}$ is the desired value at the frequency bins corresponding to $\Omega$.

### 2.3. Information-Theoretic Criteria

The above criteria do not effectively exploit the target and environmental information available through the relation in (1). Since the radar system aims to draw significant information about the target signature from the received signal, it is natural to consider the maximization of the mutual information between the amplitude of the target return and the received signal as the design objective. This metric is considered in literature, for e.g., [2], [12–16]. In particular, for the relation in (1), modelling the various components as Gaussian, the signal design involves maximizing the mutual information $I(\cdot)$,

$$I(\boldsymbol{r}, \boldsymbol{x}|\boldsymbol{\theta}) \propto \log\det\left(\boldsymbol{I}_N + \boldsymbol{\Lambda}_{\boldsymbol{\theta}}^H \left[\boldsymbol{R}_{cc} + \boldsymbol{R}_{nn}\right]^{-1} \boldsymbol{\Lambda_\theta}\boldsymbol{Q}\right), \tag{6}$$

over $\boldsymbol{Q}$, where, $\boldsymbol{Q} = E[\boldsymbol{x}\boldsymbol{x}^H]$ subject to a power constraint on $\mathrm{tr}(\boldsymbol{Q}) \leq P_x$ where $P_x$ denotes the signal power.

### 2.4. Signal-to-Interference-plus-Noise Ratio (SINR)

SINR is a classical metric that is widely used when a coherent processing of the $N$-dimensional signal in (1) is to be considered; it takes the form,

$$SINR = \frac{\left|\boldsymbol{w}^H \boldsymbol{\Lambda_\theta}\boldsymbol{x}\right|^2}{\boldsymbol{w}^H \left[\boldsymbol{R}_{cc} + \boldsymbol{R}_{nn}\right] \boldsymbol{w}}, \tag{7}$$

where $\boldsymbol{w}$ is the receive filter (a.k.a the beamforming vector). Signal design maximizing $SINR$ subject to power constraint, $||\boldsymbol{x}||_2^2 \leq P_x$ has been studied extensively; see e.g. [5, 6, 17], and the references therein.

## 3. METRIC INTEGRATION FOR RELIABILITY

Waveform design for robustness or reliability in cognitive radar is a proactive approach to ensure that the quality of probing waveforms is *satisfactory* as measured by several quality metrics (such as those discussed in Section 2). The problem thus can be formulated as an optimization problem in which several quality metrics (also referred to as objectives) are to be maximized simultaneously. The need for metric integration arises from the multitude of scenarios under which the cognitive radar is envisaged to operate and the uncertainties involved therein. In the following, we review some specific scenarios where design for reliability/robustness is instrumental.

### 3.1. Clutter Uncertainty

The clutter model used in [5, 6] assumes a stationary process with known $\boldsymbol{R}_{cc}$. However, in many situations, like in the case of dynamic scenes, the clutter process may not be stationary and/or $\boldsymbol{R}_{cc}$ may not be estimated accurately. Assuming $\boldsymbol{\theta}$ to be known, uncertainty in $\boldsymbol{R}_{cc}$ affects maximizing $I(\cdot)$ and SINR. In particular, maximizing (7) for robustness in clutter uncertainty may be formulated as

$$\begin{aligned} &\max_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{w}} \;\; \forall_{\boldsymbol{R}_{cc}}\{SINR(\boldsymbol{R}_{cc})\}, \\ &\text{s. t. } \boldsymbol{R}_{cc} = \widehat{\boldsymbol{R}}_{cc} + \boldsymbol{\Delta},\; ||\boldsymbol{\Delta}|| \leq \delta, \end{aligned} \tag{8}$$

where $\mathcal{X}$ is the constraint set of the probing signals, $\widehat{\boldsymbol{R}}_{cc}$ is the estimated covariance matrix of clutter, and $\boldsymbol{\Delta}$ denotes the uncertainty of the covariance estimation. The uncertainty is bounded by $\delta$, with $||\cdot||$ referring to an appropriate matrix norm in (8). It should be noted that (8) assumes a particular additive uncertainty model; other models driven by the scenario settings can be used instead with appropriate constraints.

### 3.2. Doppler Uncertainty

Target detection in presence of clutter can be enhanced by exploiting the Doppler shift of moving targets; this arises from the fact that the Doppler shift is a signature of the target velocity and differs from those of the clutter scatterers [5,6,18,19]. An exploitation of the Doppler warrants its knowledge at the transmitter, which, more often than not, is also a subject of investigation. In the absence of Doppler information, the SINR maximization takes the form,

$$\max_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{w}} \;\; \forall_{\boldsymbol{\theta}\in\Omega} \;\; \{SINR(\boldsymbol{\theta})\}, \tag{9}$$

where $\boldsymbol{\theta}$ is now a scalar parameter representing the normalized Doppler shift while $\Omega$ represents the possible range of Doppler (apriori information). This formulation is further developed in [5].

### 3.3. Angular Uncertainty

MIMO radars have been researched exhaustively in the past decade due to the advantages offered by the spatial degrees of freedom [1]. The angle of arrival from target transmission affects[2] the SINR and hence the design of transmit signals and receive filters in (7). In practice, the target angle is obtained from knowledge-aided methods or estimated through prescan of the environment. However, there are other situations where only the range of possible target angles are known, for example in automotive applications where the pedestrian (weak target) is in-front of a large metallic reflector (strong interference). In such situations, maximizing SINR takes the form presented in (9), with $\boldsymbol{\theta}$ now denoting the target angle. Further, $\Omega = [\theta_c - \epsilon, \theta_c + \epsilon]$ with $\theta_c$ denoting an approximate angle and $\epsilon$ indicates the level of angular uncertainty. This motivates the recourse to an angular-robust design requiring maximization of the worst-case output SINR [20].

### 3.4. Spectral Coexistence

The ever-growing demand from communication systems, and the trend towards an *utilitarian* high resolution sensing imposes significant strain on the scarce bandwidth, thereby motivating coexistence of high quality communication and sensing devices [21–24]. Assuming the communication device to be the primary user, enabling cognition at the sensing counterpart allows for exploitation of available spectrum efficiently by reusing unused bands for a limited duration locally. This is enabled by dynamic spectrum management and spectrally shaped waveforms. The cognitive radar waveform is required to have nulls in specific bands to minimize interference with systems operating in those bands [11]. The $SEM(\boldsymbol{x})$ metric of (5) can be extended to such scenarios as,

$$\min_{\boldsymbol{x}\in\mathcal{X}} \;\; \forall_{k\in[1,K]} \;\; \{\|\boldsymbol{F}(\Omega_k)\boldsymbol{x} - \boldsymbol{\Xi}\boldsymbol{y}_k\|^2\}, \tag{10}$$

where $K$ different spectra (i.e. $\{\boldsymbol{y}_k\}$) are to be *matched*.

### 3.5. Detection and Estimation— A Joint Perspective

Increasingly, a number of research works are being considered on waveform design algorithms which not only maximize the detection performance but, at the same time, are also capable of controlling the estimation accuracy of target parameters of interest. These problems naturally lend themselves to being formulated as multi-objective optimization. In [25], a constrained multi-objective optimization problem to design the spectral parameters of the OFDM waveform to

[2] $\Lambda_{\boldsymbol{\theta}}(k,k)$ could be array response of the $k$th sensor

minimize the sparse-estimation as well as enhancing detection capability is devised. Similar approaches maximizing SINR (respectively detection probability) as well as the accuracy of parameter estimation needed to determine SINR have been considered [26], [19].

## 4. ALGORITHMS FOR RELIABLE WAVEFORM DESIGN

Having discussed the scenarios warranting reliability and motivating waveforms be robust to different figures of merit, we briefly indicate some design methodologies.

### 4.1. Max-Min and Average Designs

An usual approach to reliable waveform design is maximizing the minimal performance of the system considering the various parameters. This is particularly useful when the objectives to be optimized are actually drawn from the same quality metric. For example, (9) is typically cast as,

$$\max_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{w}} \min_{\boldsymbol{\theta}\in\Omega} SINR(\boldsymbol{\theta}). \tag{11}$$

On the other hand, ascribing a prior to the unknown parameters allows us to exploit statistics of the cost function, most notably the mean. In particular, we could consider the following instead of (12),

$$\max_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{w}} E_{\boldsymbol{\theta}}\left[SINR(\boldsymbol{\theta})\right]. \tag{12}$$

### 4.2. Pareto-Optimal Waveform Design

Instead of a single objective optimization, it is possible to have multiple figures of merit for designing $\boldsymbol{x}$. This leads to the multi-objective optimization function of the form,

$$\max_{\boldsymbol{y}\in\mathcal{Y}}. \quad \boldsymbol{g_\theta}(\boldsymbol{y}) = [g_{1,\boldsymbol{\theta}}(\boldsymbol{y}), g_{2,\boldsymbol{\theta}}(\boldsymbol{y}), \ldots, g_{M,\boldsymbol{\theta}}(\boldsymbol{y})], \tag{13}$$

where $g_{k,\boldsymbol{\theta}}(\cdot)$ denotes the $k$th metric, $\boldsymbol{y}$ represents the optimization variables (typically the waveform $\boldsymbol{x}$, but can include others like the beamforming vector) drawn from the set $\mathcal{Y}$. When the different objectives are in conflict, the optimization in (13) does not have a global optimum [27]. In such situations, Pareto optimality, the state of allocation of resources (or the optimization variables) in which it is impossible to improve any one quality metric without making at least another quality metric worse off is usually considered.

An overview of approaches towards estimating some points of the Pareto boundary in presented in [27]. We now present a different approach that allows for the incorporation of multiple objective optimization in radar domain and then provide an algorithm for Pareto-optimal waveform design.

#### *4.2.1. Iterative approach for Pareto-optimal design*

Let $z$ be the variable indicating the different objectives. Depending on the constraints, $z$ could be discrete or continuous; (13) is as example of the former, while (11) is an example of the latter. In the following, without loss of generality, we consider the case of discrete $z$ (the results can be extended to its continuous counterpart). It can then be argued that, (13) is equivalent to the following optimization problem,

$$\begin{aligned} &\max_{\boldsymbol{y}} \ \ \forall_{z\in[1,M]}\{\boldsymbol{h_\theta}(\boldsymbol{y},z)\} \\ &\text{s. t.} \ \ \boldsymbol{y}\in\mathcal{Y} \end{aligned} \tag{14}$$

where $\boldsymbol{h_\theta}(\boldsymbol{y},z) = g_{z,\boldsymbol{\theta}}(\boldsymbol{x}),\ z\in[1,M]$. The following proposition, proved in [28], provides an iterative algorithm for obtaining Pareto-optimal solutions of (14).

*Proposition 1:* Assuming that all Pareto-optimal solutions of (14) are finite, a Pareto-optimal solution of (14) can be obtained using the following iterative approach. Given $\boldsymbol{y}^{(t)}\in\mathcal{Y}$, $(t\geq 0)$, obtain $\boldsymbol{y}^{(t+1)}$ as the solution to the following (single-objective) optimization problem,

$$\begin{aligned} &\max_{\boldsymbol{y}^{(t+1)}} \quad \min_{z} \left\{ \frac{\boldsymbol{h_\theta}\left(\boldsymbol{y}^{(t+1)},z\right)}{\boldsymbol{h_\theta}\left(\boldsymbol{y}^{(t)},z\right)} \right\} \\ &\text{s. t.} \quad \boldsymbol{y}^{(t+1)}\in\mathcal{Y}. \end{aligned} \tag{15}$$

**Remark.** It is worth noting that a *max-min design is always Pareto-optimal.* In fact, it is not difficult to verify that a max-min design is a convergence point of the iterative approach in Proposition 1. Examples of max-min can be found, e.g., in [5] and [6].

## 5. CONCLUSIONS

An overview of the reliability aspects of the emerging cognitive radio paradigm is provided and appropriate waveform design metrics are highlighted. Algorithms for waveform design are subsequently described including the max-min formulation as well as a multi-objective optimization incorporating relevant design metrics.

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