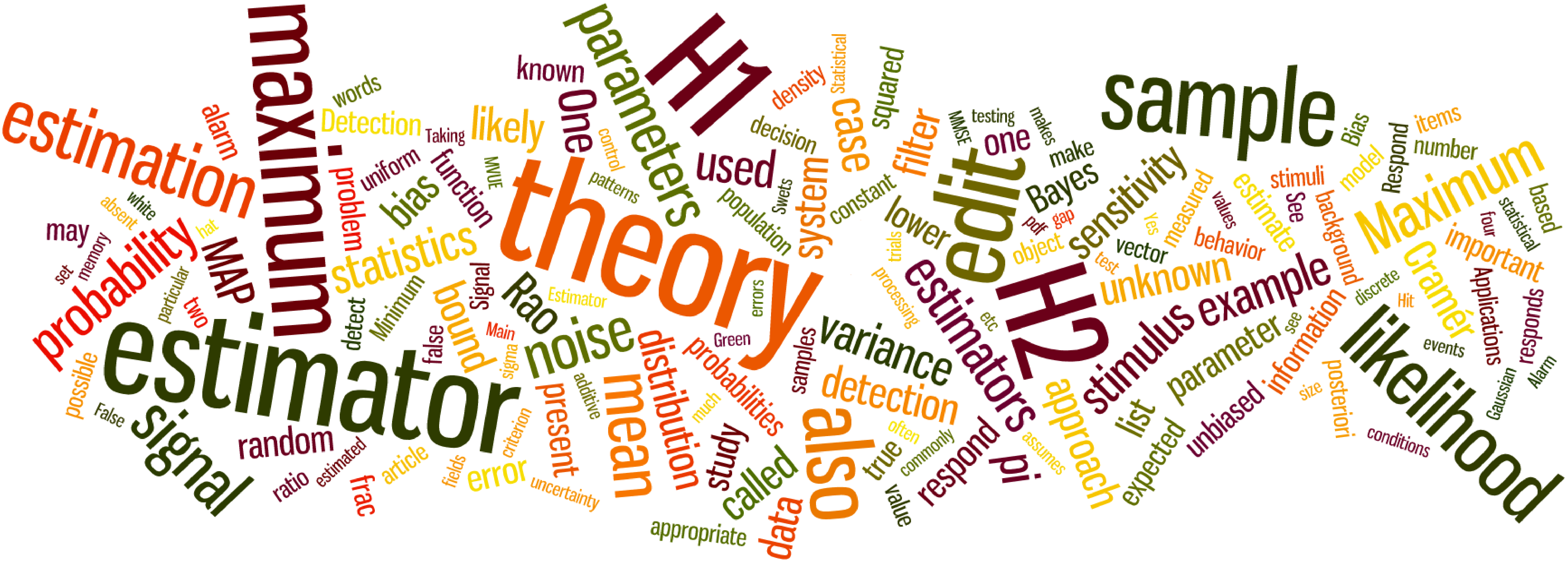



# Detection and Estimation Theory
## Lectures 20

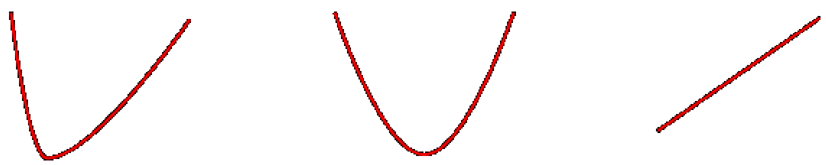
Mojtaba Soltanalian- UIC
msol@uic.edu
http://msol.people.uic.edu

# Optimization for Detection and Estimation

## Mathematical Optimization

# Optimization for Detection and Estimation

Convex vs Non-Convex



# Convex Optimization

# Optimization for Detection and Estimation

## Non-Convex Optimization

# Other
# “Local Algorithms”

$f(\theta)$

the function to be minimized

The common feature of the algorithms in this part is that they all monotonically decrease the function at each iteration

$$f\left(\theta^{i+1}\right) \leq f(\theta^{i}) \quad \text{for } i = 0, 1, 2, \ldots .$$

# Cyclic Minimizer

To describe the main idea of this type of algorithm in its simplest form, let us partition $\theta$ into two subvectors

$$\theta = \begin{bmatrix} x \\ y \end{bmatrix}.$$

Then the generic iteration of a cyclic algorithm for minimizing $f(x, y)$ will have the following form:

$y^0 = \text{given}$

For $i = 1, 2, \ldots$ compute:

$$x^i = \arg\min_x f(x, y^{i-1})$$

$$y^i = \arg\min_y f(x^i, y).$$

$$f(x^i, y^i) \le f(x^i, y^{i-1}) \le f(x^{i-1}, y^{i-1})$$

# Majorization Technique

Majorization-minimization (MaMi) is an iterative technique that can be used for obtaining a solution, i.e. a stationary point, of the general minimization problem

$$\min_{\mathbf{z}} \widetilde{f}(\mathbf{z}) \quad \text{subject to} \quad c(\mathbf{z}) \leq 0 \tag{6}$$

where $\widetilde{f}(.)$ and $c(.)$ might be non-convex functions. Each iteration (say the $l^{th}$ iteration) of MaMi consists of two steps (see Fig. ):

- Majorization Step: Finding $p^{(l)}(\mathbf{z})$ such that its minimization is simpler than that of $\widetilde{f}(\mathbf{z})$, and $p^{(l)}(\mathbf{z})$ majorizes $\widetilde{f}(\mathbf{z})$:

$$p^{(l)}(\mathbf{z}) \geq \widetilde{f}(\mathbf{z}), \quad \forall \mathbf{z} \quad \text{and} \quad p^{(l)}(\mathbf{z}^{(l-1)}) = \widetilde{f}(\mathbf{z}^{(l-1)}) \tag{7}$$

with $\mathbf{z}^{(l-1)}$ being the value of $\mathbf{z}$ at the $(l-1)^{th}$ iteration.

- Minimization Step: Solving the optimization problem,

$$\min_{\mathbf{z}} p^{(l)}(\mathbf{z}) \quad \text{subject to} \quad c(\mathbf{z}) \leq 0 \tag{8}$$

to obtain $\mathbf{z}^{(l)}$.