A Transform Domain LMS Adaptive Filter With Variable Step-Size

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***Abstract*—In this letter, we introduce a new transform domain (least mean square) LMS algorithm with variable step. The existing approaches use different time-variable step-sizes for each filter tap. The step-sizes are time-variable due to the power estimates of each transform coefficient. In our new approach, for each step-size we define a local component that is given by the power normalization, and a global component that is the same for each filter coefficient. We will show here that if the global component is also made time-variable, depending on the output error, the speed of convergence can be significantly improved.**

***Index Terms*—Adaptive algorithms, discrete cosine transform (DCT), transform domain LMS, variable step-size.**

I. Introduction

IN ADAPTIVE signal processing applications, the least mean square (LMS) algorithm [1] is commonly used due to its simplicity. For highly correlated input signals, the speed of convergence of the time-domain LMS algorithm degrades dramatically. As an alternative solution, modifications of the LMS algorithms with variable step size as well as transform domain LMS (TDLMS) algorithms have been developed [2]–[9].

In the case of TDLMS, an input signal is transformed by the use of an orthogonal transform and the filter coefficients are updated independently. In the existing approaches, the step-sizes are usually considered time-variable due to the use of the power estimates of the transform coefficients, but when the power estimates become constants, the step-sizes also become constants. Therefore, when the input signal is stationary, the step-sizes of each filter tap are time-variable only on the early stages of the adaptation. However, there are no known TDLMS algorithms that use the output error in the update of the step-size. In this letter, we have developed a modification of the TDLMS algorithm which has the following feature: we define for each step-size a local component depending on the power normalization and a global component that is the same for each filter tap. The global component is also time-variable and depends on the output error so that the speed of convergence is significantly improved.

II. Existing Approaches

The block diagram of the adaptive system identification using the TDLMS filter is shwon in Fig. 1, where the block denoted by T_N represents the transform matrix applied to the block of the input signal $x(n)$, $\mathbf{s}(n)$ is the vector of the transform coefficients, $\hat{y}(n)$ and $y(n)$ are the output signals of the adaptive filter and the unknown system, respectively, $v(n)$ is the output noise, and $d(n)$ and $e(n)$ are the desired signal and the error signal, respectively.

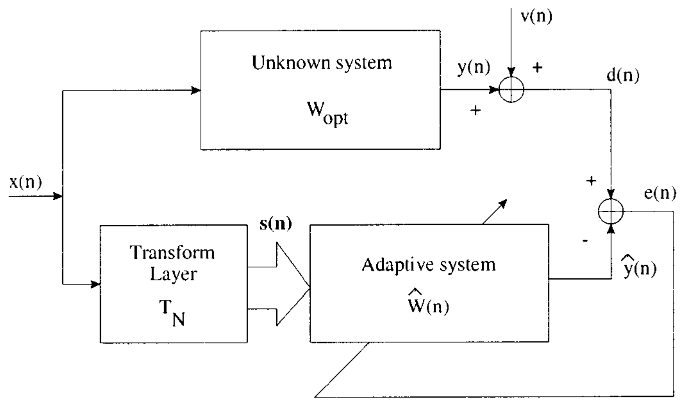

Fig. 1. Block diagram of system identification using the transform domain adaptive filtering.

The desired signal and the error signal are computed by

$$d(n) = \mathbf{w}_{opt}^T \mathbf{x}(n) + v(n); \quad e(n) = d(n) - \hat{\mathbf{w}}^T(n)\mathbf{s}(n).$$

The update of each adaptive filter coefficient is given by

$$\hat{w}_i(n+1) = \hat{w}_i(n) + \frac{\mu}{\epsilon + \sigma_i^2(n)} e(n) s_i(n) \tag{1}$$

where $\sigma_i^2(n)$ is the power estimate of the ith transform coefficient $s_i(n)$, $\hat{w}_i(n)$ is the ith coefficient of the adaptive filter, and ϵ is a small constant that eliminates the overflow when the values of $\sigma_i^2(n)$ are small.

For computing the value of $\sigma_i^2(n)$ the exponential weighted method is usually used as follows:

$$\sigma_i^2(n) = \beta \sigma_i^2(n-1) + (1-\beta)|s_i(n)|^2 \tag{2}$$

where $\beta \in [0, 1]$ is the forgetting factor.

In another approach [8], the following formula for the step-size update is used:

$$\mu_i(n+1) = \beta \mu_i(n) + \gamma(1-\beta)\left(\frac{1}{\epsilon + \frac{1}{M}\mathbf{s}_i(n)^T \mathbf{s}_i(n)}\right)$$

where $\mathbf{s}_i(n) = [s_i(n), s_i(n-1), \ldots, s_i(n-M+1)]^T$ is the vector of the past M values of the ith transform coefficient, and $s_i(n)$, $\beta \in [0, 1]$, $\gamma \in [0, 1]$, and $0 < \epsilon \ll 1$ are some constant parameters.

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III. New Algorithm

There have been many developments in the field of transform domain LMS adaptive filtering [5]–[9], but the algorithms proposed so far (to the best of our knowledge) do not use the output error in the update of the step-size. Here, we propose a new algorithm that uses the output error in order to update the step-size of each filter tap resulting in significant improvement of the convergence speed.

In the existing methods, the step-size of the ith coefficient is given by [see (1)]

$$\mu_i(n) = \frac{\mu}{\epsilon + \sigma_i^2(n)}. \tag{3}$$

In (3), the numerator μ can be viewed as the global component of the step-size, since it is the same for each coefficient. The denominator of (3) can be viewed as the local component of the step-size, since it depends on the power estimate $\sigma_i^2(n)$ of the corresponding transform coefficient.

In the approaches proposed so far, only the local component is time-variable. Also, in the new algorithm, the global component is variable and depends on the output error as follows:

$$A(n) = \alpha\mu(n) + \frac{\gamma}{L} \sum_{i=n-L+1}^{n} e(i)^2 \tag{4}$$

$$\mu(n+1) = \begin{cases} A(n), & \text{if } n = kL, \text{ and } \\ & A(n) \in (\mu_{\min}, \mu_{\max}) \\ \mu_{\max}, & \text{if } n = kL, \text{ and } A(n) \geq \mu_{\max} \\ \mu_{\min}, & \text{if } n = kL, \text{ and } A(n) \leq \mu_{\min} \\ \mu(n), & \text{if } n \neq kL \end{cases} \tag{5}$$

where $k = 1, 2, 3, \ldots$.

Therefore, the step-size of the ith coefficient becomes

$$\mu_i(n) = \frac{\mu(n)}{\epsilon + \sigma_i^2(n)}. \tag{6}$$

For computing the value of $\sigma_i^2(n)$ we have used (2). Hence, in the case of the new TDVSS algorithm, the update of each filter coefficient is given by

$$\hat{w}_i(n+1) = \hat{w}_i(n) + \frac{\mu(n)}{\epsilon + \sigma_i^2(n)} e(n) s_i(n) \tag{7}$$

where the notations are the same as for the standard TDLMS algorithm and $\mu(n)$ is given by (4) and (5).

The behavior of the new TDVSS algorithm can be described as follows: for a number of L consecutive iterations (the test interval of length L), the global component $\mu(n)$ is constant, and the new algorithm behaves as a standard TDLMS. At the end of the test interval the average of the past L squared values of the error is computed, and $\mu(n)$ is updated using (4) and (5). In this way, when the output error has higher level, also the step-size has higher value, such that the convergence speed is increased. When the adaptive filter goes toward the steady-state, the error will have smaller values and, therefore, the step-size is decreased in order to achieve a desired level of the misadjustment.

TABLE I
Computational Complexity of TDLMS, DCT-LMS, and TDVSS

Algorithm	TDLMS	DCT-LMS	TDVSS
Mult./Div.	$6N+1$	$5N+MN$	$6N+4$
Add./Sub.	$3N$	$3N+MN$	$3N+2$

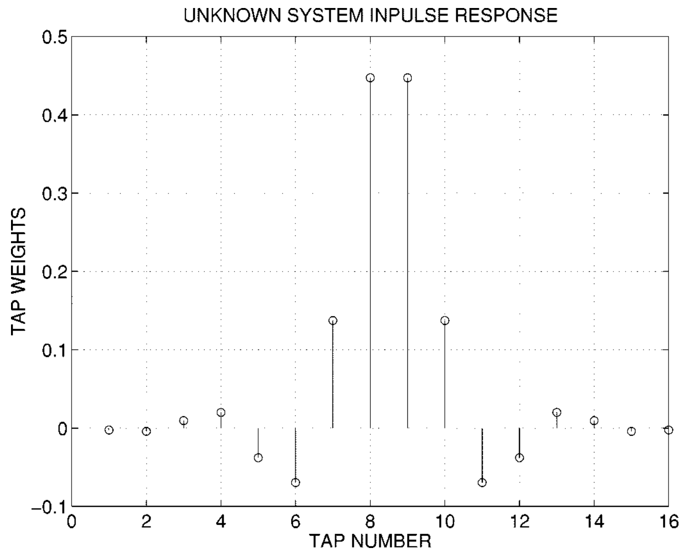

Fig. 2. Unknown system impulse response.

The value of the parameter L in (4) and (5) must be sufficiently large so that the behavior of the error can be obtained by the use of the average function, and also the value of L must be small in order to have enough step-size updated during the training. In all our simulations, we have used $L = 10$. Usually, the parameter γ in (4) has a small value, and may be chosen to meet the misadjustment requirements.

The computational complexity (without transform T_N) of the conventional TDLMS, discrete cosine transform (DCT)-LMS [8] and the new TDVSS are presented in Table I, and we can see that the computational complexity of the new algorithm is comparable with that of the TDLMS.

IV. Simulations and Results

We tested our new algorithm in the system identification framework. The unknown system coefficients are shown in Fig. 2, and the transform used was the DCT. The input signal was

$$x(n) = 1.79x(n-1) - 1.85x(n-2) + 1.27x(n-3) - 0.41x(n-4) + \nu(n)$$

where $\nu(n)$ is a white Gaussian random signal with zero mean and variance $\sigma_\nu^2 = 0.148\,17$. The eigenvalue spread ratio of the input signal $x(n)$ is 944.67. The signal to noise ratio at the output of the unknown system was 50 dB, and all the simulations were obtained by averaging 200 Monte-Carlo simulations of the algorithms. The new algorithm is compared with the plain LMS, VSS [3], MVSS [4], common TDLMS, and DCT-LMS [8]. The parameters of all algorithms are given in Table II, and were chosen such that the steady-state misadjustments are comparable. The misadjustments of all the algorithms are presented in Table III.

TABLE II
PARAMETERS OF THE COMPARED ALGORITHMS

LMS	VSS	MVSS
$\mu = 5 \cdot 10^{-3}$	$\gamma = 10^{-3}$ $\mu_{min} = 5 \cdot 10^{-3}$ $\alpha = 0.97$ $\mu_{max} = 5 \cdot 10^{-2}$	$\mu_{max} = 5 \cdot 10^{-2}$ $\alpha = 0.97$ $\beta = 0.99$ $\mu_{min} = 5 \cdot 10^{-3}$ $\gamma = 8 \cdot 10^{-4}$
TDLMS	**DCT-LMS**	**TDVSS**
$\epsilon = 2.5 \cdot 10^{-2}$ $\beta = 0.9$ $\mu = 4.7 \cdot 10^{-3}$	$M = 10$ $\gamma = 2 \cdot 10^{-3}$ $\beta = 0.9985$ $\epsilon = 8 \cdot 10^{-4}$	$\mu_{max} = 5 \cdot 10^{-2}$ $\alpha = 0.99$ $L = 10$ $\beta = 0.9$ $\mu_{min} = 4.7 \cdot 10^{-3}$ $\gamma = 10^{-3}$ $\epsilon = 2.5 \cdot 10^{-2}$

TABLE III
MISADJUSTMENTS OF THE COMPARED ALGORITHMS

Algorithm	LMS	VSS	MVSS
$M = \frac{\xi_{ex}}{\xi_{min}}$	4.5274%	4.8889%	4.8859%
Algorithm	TDLMS	DCT-LMS	TDVSS
$M = \frac{\xi_{ex}}{\xi_{min}}$	4.2141%	4.1356%	4.2141%

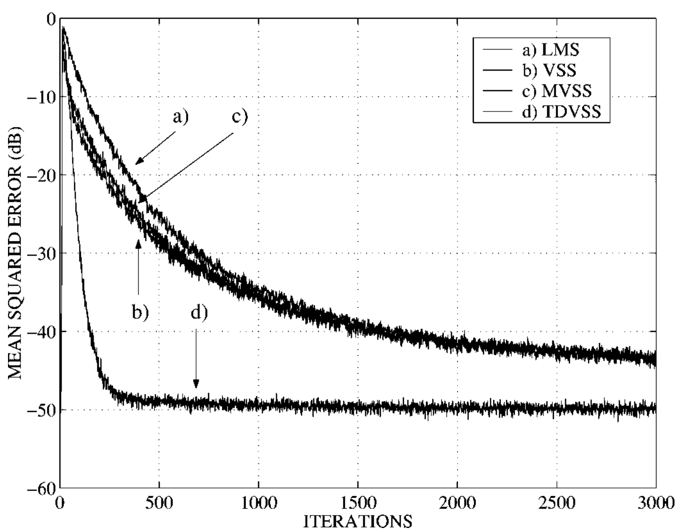

Fig. 3. Comparison between LMS, VSS, MVSS, and TDVSS.

In Fig. 3, we present the behavior of the LMS, VSS, MVSS, and TDVSS algorithms. As expected, the time domain implementations do not perform well when the input signal is highly correlated. Fig. 4 shows the comparison between the TDLMS, DCT-LMS, and TDVSS algorithms. One can see from Fig. 4 that the use of the output error in the update of the time-variable step-size can significantly improve the speed of convergence for the transform domain implementation of the LMS algorithm.

V. CONCLUSIONS

In this letter, we have introduced a new transform domain LMS algorithm with a time-variable step-size. The difference between the new algorithm and the existing approaches known so far is that the step-size depends also on the output error. Using such update for the step-size, the computational complexity of the new TDVSS algorithm and the common TDLMS algorithm are comparable, but the speed of convergence is significantly increased.

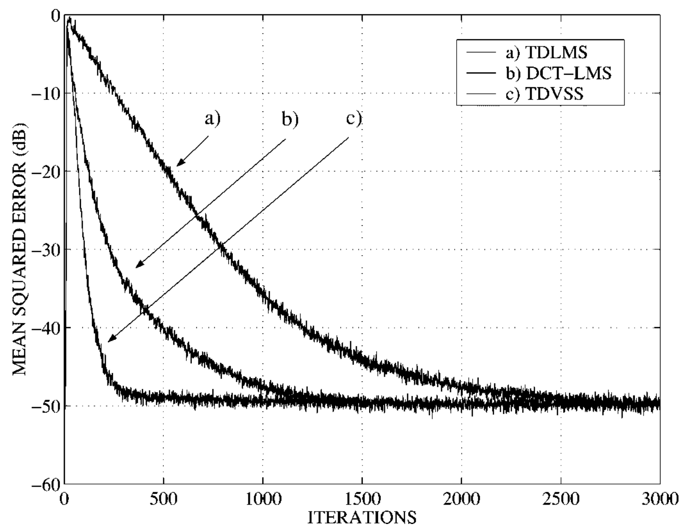

Fig. 4. Comparison between TDLMS, DCT-LMS, and TDVSS.

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